SLAPI v1.1

Documentation

PITAGORAS

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"**Just intervals** are intervals made from the ratio of small whole numbers. The only interval that is just in the equal-tempered scale is the octave, 2/1…*just scales* are based entirely on such small whole-number ratios."

Gareth Loy, *Musimathics: The Mathematical Foundations of Music*[1]

"**Equal Temperament**: the practice of dividing the octave into an equal number of parts, or of making up a scale from equal-sized steps."

Kyle Gann, *My Idiosyncratic Reasons for Using Just Intonation*[2]

"It has long been noted that the common harmonious, consonant intervals are more or less accurately represented by frequency ratios involving small integers."

Easley Blackwood, *The Structure of Recognizable Diatonic Tunings*[3]

"A musical system that calls for harmonically pure intervals at all times is called **Just intonation**."

Ross W. Duffin, *How Equal Temperament Ruined Harmony*[4]

Cover image: Detail from Franchino Gaffurio (1451-1522) *Theorica musicae*.[5]

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[1] Loy 2006, 43.
[2] Gann 2006d.
[3] Blackwood 1986, 5
[4] Duffin 2007, 33.
[5] Wikimedia Commons - http://en.wikipedia.org/wiki/File:Gafurino.jpg

Contents

1. Introduction

   String Length and Pitch Interval (SLAPI) is a just interval player that provides the user with an easy way to produce just intervals and compare them with their equal-tempered counterparts on the modern piano. Just intervals, intervals based on small-order whole number ratios, may be found in the lower reaches of the harmonic series[6] and in historical tuning systems such as Pythagorean and just intonation.[7] SLAPI will be of interest to music students who are encountering the harmonic series or just intonation for the first time. It may also be of interest to K-12 math teachers who are looking for a simple way to sonify fractions, that is, to demonstrate what fractions "sound like" in the musical domain.

   I created SLAPI so my students, mostly undergraduate music majors studying music theory, could freely explore the sonic signatures of the pitch intervals implied by the harmonic series. Although my theory students come to me with a good understanding of traditional tonal scales, intervals, and chords, they know very little about the harmonic series and how it may be used as a model for tuning intervals and creating scales. I originally set out to create a virtual monochord, a single-stringed instrument used by medieval theorists to study the laws of acoustics. However, after obtaining feedback from some of my students on early prototypes of that application, I discovered that I needed to make something much simpler: SLAPI.

   A basic knowledge of tonal theory fundamentals, specifically pitch and interval notation, is required to make good use this program. An understanding of how to add, subtract, multiply and divide fractions is also required.[8] In America, theory fundamentals are usually taught in high school AP Music Theory courses and in the first semester of university-level courses of music theory.[9] Equivalent knowledge through similar theory courses, or through the study of jazz theory, should also be adequate. For students who would like to learn more about the harmonic series and just intonation, I recommend Bain 2006 and Gann 2006c, respectively. If you are looking for more ratios to explore, consult Gann 2006a or Partch 1974. My favorite comprehensive introduction to just intonation is Doty 2002. I also highly recommend Loy 2006 and Fauvel 2003. If you are looking for a software application that will allow you to experiment with various tunings and temperaments I recommend Manuel Op de Coul's *Scala*.[10]

2. Computer Requirements

   SLAPI runs as a standalone application on Macintosh and Windows computers. It was created with Cycling 74's Max/MSP.[11] The approximate requirements for each platform are given below:

   Macintosh OS X: Mac PPC or Intel machine running OS X 10.4.11 or later, 1 GB RAM.

   Windows: Windows XP, Vista, or Windows 7 machine and 1 GB RAM.

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[6] Doty (2002) defines the harmonic series as "the series of all integer (positive whole-number) multiples of some frequency."
[7] See Gann 2006b for more information.
[8] See Gann 2006c for more information.
[9] Kostka and Payne 2008, 3-23.
[10] http://www.huygens-fokker.org/scala/
[11] http://cycling74.com/products/maxmspjitter/

3. Feature List

Fig. 1 shows SLAPI's interface and default settings.
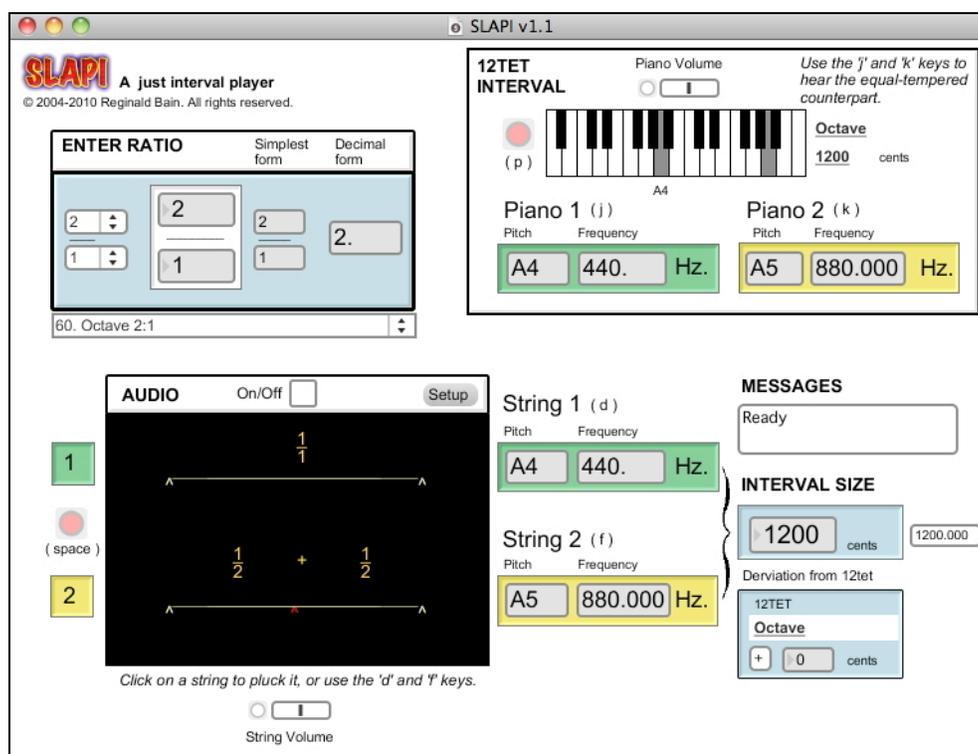

Fig. 1. SLAPI's interface and default settings.

Enter a ratio (1:1 to 2:1) and SLAPI will:

1. Display the ratio's simplest form and decimal form.

2. Draw two proportional string length diagrams (not to scale) named *String 1* and *String 2*. String 2 graphically portrays the string length division required to produce the ratio.

3. Play the pitch interval associated with the ratio using a string sound.

4. Display the interval's component pitches and frequencies, its size in cents, and its deviation from twelve-tone equal temperament (12tet) in +/- cents.

5. Determine the interval's 12tet counterpart, that is, its closest intervallic equivalent in 12tet. The 12tet counterpart's component pitches are named *Piano 1* and *Piano 2*. SLAPI will display pitch and frequency information for the 12tet interval along with its traditional interval name and size in cents, and will graphically portray it on a two-octave piano keyboard.

6. Play the 12tet counterpart using a piano sound.

4.   Interface Overview

SLAPI's interface has six area as shown in Fig. 2. The name of each numbered area is given in the legend below Fig. 2.
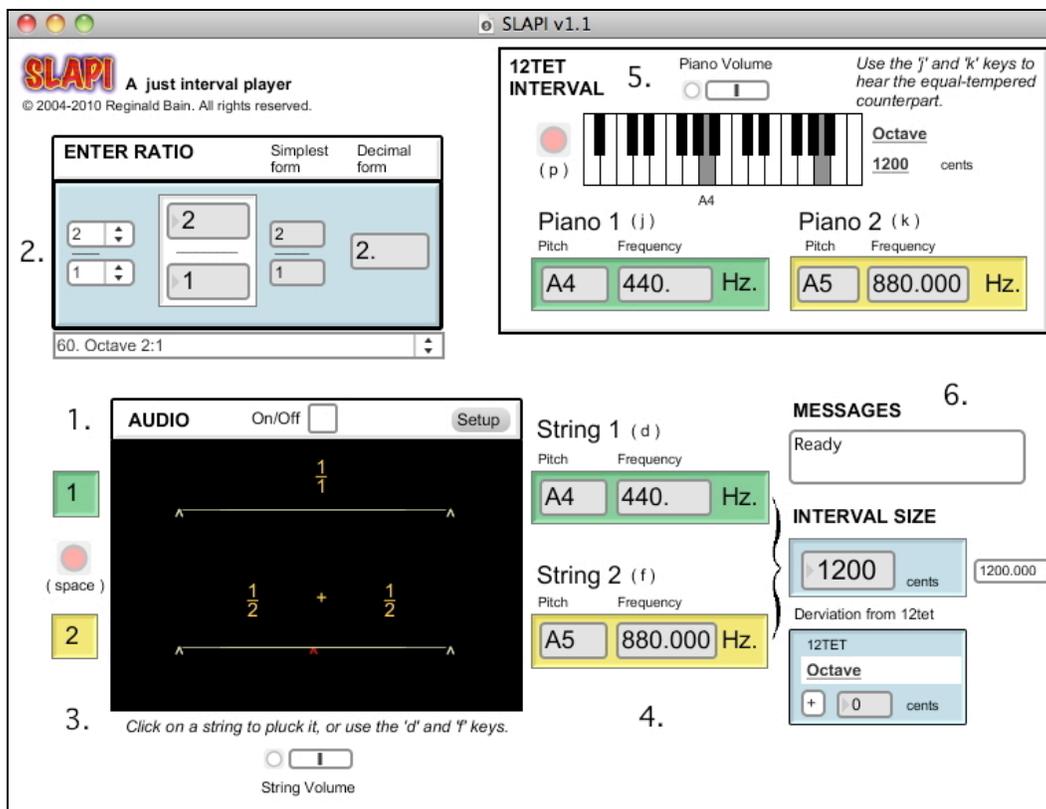

Fig. 2. SLAPI's six interface areas.

Legend

1. Audio Control Bar

2. Enter Ratio

3. String Length Diagrams

4. Interval Information

5. 12tet Interval Information

6. Messages

These areas are described in detail in sections 7-12.

5. Getting Started

Launch the application and follow these four steps to get started using SLAPI.

1. **Turn audio on**
   Click the On/Off toggle button in the Audio Control Bar to turn the audio on (Fig. 3).

Fig. 3. The Audio Control bar.

If your audio hardware and software are correctly configured, a hybrid string-piano tone will sound to confirm that the audio is on. If you don't hear this sound, click the Setup button and refer to section 14. Troubleshooting of the documentation.

2. **Enter a pitch interval ratio**
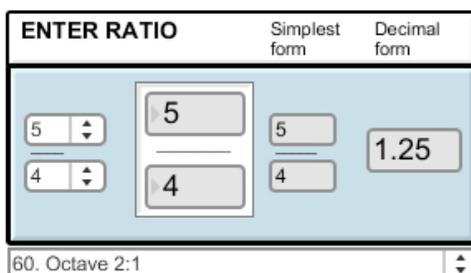   Enter a ratio in the Enter Ratio area (Fig. 4).

Fig. 4. The Enter Ratio area.

The two large number boxes (Fig. 4, middle) define the *current ratio*. You can set the current ratio by typing positive whole numbers into the large boxes, or by using the two *ratios menus* (Fig. 4, left). SLAPI will automatically calculate the simplest form and decimal form. Any change to the current ratio will cause all of SLAPI's display and playback elements to be updated immediately. For more information, see section 8. Entering Ratios.

3. **Press the space bar, or press the 'd' and 'f' keys, to hear the just interval**
   Press the *space bar* to hear the pitch interval associated with the current ratio. To hear the interval's component pitches *String 1* and *String 2*, press the 'd' or 'f' keys, respectively.

4. **Press the 'p' key, or press the 'j' or 'k' keys, to hear the 12tet counterpart**
   Press the 'p' key to hear the interval's 12tet counterpart. To hear the 12tet counterpart's component pitches *Piano 1* and *Piano 2*, press the 'j' or 'k' keys, respectively.

6.   Interval Notation

Ratios are notated using the form:

$$\frac{x}{y},$$

where *x* is the *numerator* (top number) and *y* is the *denominator* (bottom number), or using the equivalent notations *x:y* and *x/y*. For example, the ratio for the *just perfect fifth* (just P5) may be notated:

$$\frac{3}{2},$$

3:2, or 3/2.

SLAPI is primarily designed to play *ascending* pitch intervals in the A4–A5 octave.[12] These intervals may be represented by interval ratios between 1:1 and 2:1, inclusive. An ascending interval is defined as pitch interval whose ratio takes the form *x:y*, where $x > y$. The string length associated with these intervals is represented by reciprocal ratio *y:x*.[13]

The pitch a *just P5* above A4 (440 Hz.) is E5. It's frequency may be calculated as follows:[14]

$$660 \text{ Hz.} = 440 \text{ Hz.} * \frac{3}{2}.$$

It should also be mentioned that the class of ratios 6:4, 9:6, 12:8, etc. are equivalent to 3:2, that is, they are reducible to it as demonstrated below:

$$\frac{6}{4} = \frac{3}{2} * \frac{2}{2}, \quad \frac{9}{4} = \frac{3}{2} * \frac{3}{3}, \quad \frac{12}{8} = \frac{3}{2} * \frac{4}{4}, \quad etc.$$

Thus, the ratio 3:2 is said to be the *simplest form* of this class of ratios.[15] The *decimal form* is obtained by dividing a ratio's numerator by its denominator. For example, the decimal form of 3:2 is 1.5. SLAPI rounds decimal equivalents to three decimal places.

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[12] SLAPI employs the pitch notation suggested by the Acoustical Society of America where C4 is middle C.
[13] The reciprocal of any number *n* is 1/*n*.
[14] The general formula is: $f = 440$ Hz. * *x/y*. For more information, see Gann 2006c.
[15] For more information about *ratios* and *fractions*, visit Wolfram 2010 at: <http://mathworld.wolfram.com>.

7.   String Length Diagrams

To provide an appropriate context for the study of just pitch intervals, SLAPI employs two string length diagrams (not to scale) named *String 1* and *String 2* (Fig. 4).
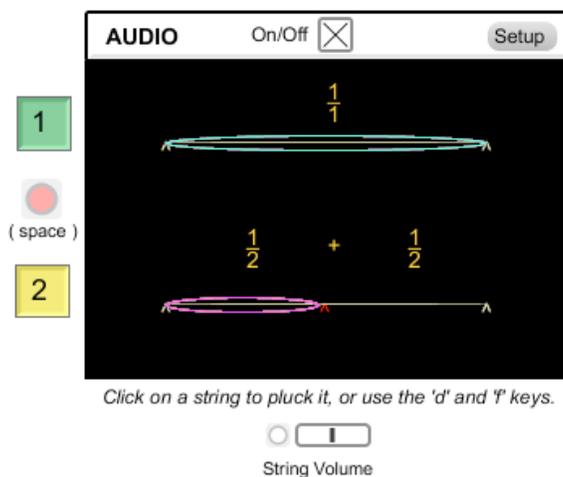

Fig. 4. String length diagrams for the just ratio 2:1.

String 1 is of *unit length* 1:1. When "plucked," it produces the pitch A4 with a frequency of 440 Hz. String 2 has the same length, pitch, and frequency as String 1 but adds a red movable bridge. Enter the ratio

$$\frac{x}{y},$$

and SLAPI will partition String 2 into two segments. The length of the left segment of String 2 is represented by the reciprocal ratio

$$\frac{y}{x}.$$

The right segment's length is represented by:

$$1 - \frac{y}{x}.$$

For the interested reader, the general equation for the fundamental frequency of vibration f_1 of a string with length L, tension T, and density d (mass per unit length) is:[16]

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{d}}$$

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[16] Backus 1977, 59. In SLAPI, the tension and density of the strings are assumed to be fixed values so that only the proportional lengths of strings need be taken into account.

When String 2's left segment is sounded against String 1, the pitch interval associated with the current ratio is sounded.[17] To hear the interval, click on the round red button to the left of the string length diagrams (Fig. 6) or press the 'space bar' on the computer keyboard. For more information about playback, see section 12. Playback Features.

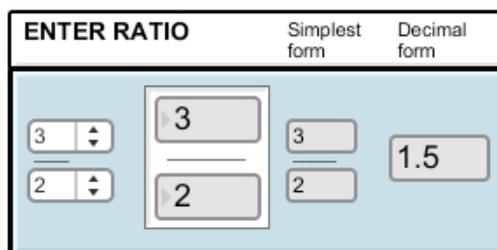Fig. 5 shows the Enter Ratio area with the current ratio set to the just P5.

Fig. 5. The current ratio set to the just P5.

Fig. 6 shows the corresponding string length diagrams during playback.
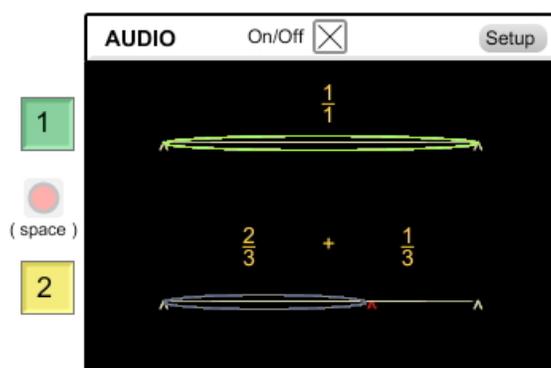

Fig. 6. String length diagrams for the just P5.

Notice that the ratio associated with the left segment of String 2 is 2:3 (the reciprocal of the current ratio 3:2), whereas the ratio associated with the right segment of String 2 is:

$$1 - \frac{2}{3} = \frac{1}{3}.$$

Finally, it should be mentioned that the frequency and amplitude of the randomly-colored *string animations* are not to scale. The animations are provided to give the general impression that the strings are "vibrating."

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[17] Please note that String 2's right segment cannot be played. If you want to hear the pitch associated with this ratio you will need to enter the appropriate ratio (its reciprocal) in the Enter Ratio area.

8.   Entering Ratios

The Enter Ratio area is shown again in Fig. 7, this time, with the *current ratio* set to the Pythagorean major 3rd 81:64.
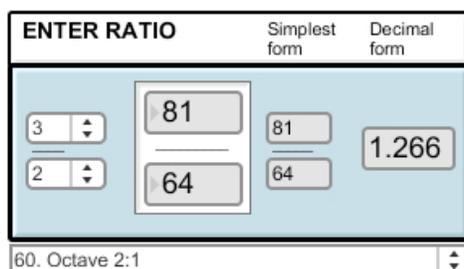

Fig. 7. The current ratio is set to the Pythagorean major third.

In Fig. 7, the notice how the two small *ratio menus* still read 3:2. This is because 81:64 was typed directly into the *current ratio number boxes*. The *current ratio* is displayed in these two large boxes, so entering 81:64 overrides the values displayed in the ratio menus. These current ratio boxes allow you to enter ratios with numerators and denominators from 1 to 999. The ratio menus, on the other hand, are designed to provide a fast way to enter ratios with numerators and denominators from 1-32. Ratios also must be less than or equal to 2:1, and greater than or equal to 1:2. Finally, it should be mentioned here that SLAPI does not display string length diagrams for intervals less than 1:1. At the bottom of Fig. 7, note the *enter ratio menu*. It contains 60 preprogrammed pitch intervals for you to explore. See Appendix 1 for more information.

Current Ratio Number Boxes

Fig. 8 shows the current ratio number boxes again, this time with the numerator in the process of being edited.
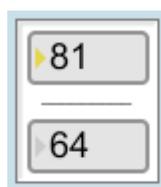
Fig. 8. The current ratio number boxes.

SLAPI offers three different ways to change the numerator and denominator of the current ratio:

1. **Type a number into a box**. A yellow triangle will appear to signal that the box is open for editing. Press the *return* (or *enter*) key when you are finished editing a number.
2. **Drag a number with the mouse** to change its value.
3. **Select a number**, then **use the up/down cursor arrow keys** to change its value.

When a change is made to the current ratio, SLAPI immediately updates all playback and display

elements.

Ascending vs. Descending Pitch Intervals

As mentioned before in section 6, if the current ratio's numerator is greater than its denominator (e.g., 3:2, 4:3, 5:3, etc.) SLAPI assumes that the ratio represents an *ascending pitch interval*. If the current ratio's numerator is less than its denominator (e.g., 2:3, 3:4, 3:5, etc.), SLAPI assumes that the ratio represents a *descending pitch interval*. SLAPI can play, and can display pitch, frequency, and interval size information for descending intervals. However, it cannot create string length diagrams for ratios less than 1:1.

9. Interval Information

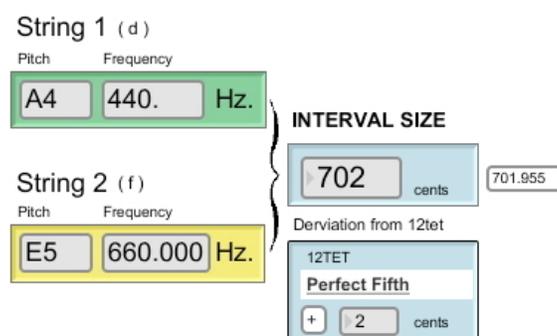The Interval Information area is shown in Fig. 9.

Fig. 9. The Interval Information area.

The values shown are again those for the just P5. Pitch and frequency information for String 1 is given in the green box. Pitch and frequency information for String 2's left segment are shown in yellow box. The frequency of String 2's left segment is calculated by multiplying String 2's base frequency (440 Hz.) by the just ratio, in this case, 3:2:

$$440 \text{ Hz.} * \frac{3}{2} = \frac{1320}{2} = 660 \text{ Hz.}$$

SLAPI calculates the size of the pitch interval in cents (c) using the following conversion formula:[18]

$$c = 1200 \log_2 \frac{f_1}{f_2}, \tag{1}$$

where f_1 is the frequency of String 1 and f_2 is the frequency of String 2's left segment. This value

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[18] Campbell and Greated 1987, 79. The *cent* is a logarithmic unit of pitch interval measurement that is based 12tet. Given that there are 1200 cents in an octave, each 12tet semitone measures 100 cents in size. Cents are frequently used to compare the sizes of intervals in different tunings. Given two frequencies f_1 and f_2, the size in cents of the pitch interval formed between them is given by (1).

is displayed in the light-blue *interval size* box and rounded to the nearest cent so that it is easy to memorize. For those who require greater precision, the size of the pitch interval in cents is also given to three decimal places in the small number box to the right.

Fig. 10 shows the Deviation from 12tet sub-area.

Fig. 10. A just P5 is equal to a 12tet P5 plus 2 cents.

It displays the pitch interval's deviation in +/- cents from its 12tet counterpart. A "+" indicates that the interval is larger than the 12tet counterpart, whereas a "-" indicates that the interval is smaller than the 12tet counterpart. This is one traditional way to describe the discrepancy in size between pitch intervals.

10. 12tet Interval Information

The 12tet Interval Information area is shown in Fig. 11.
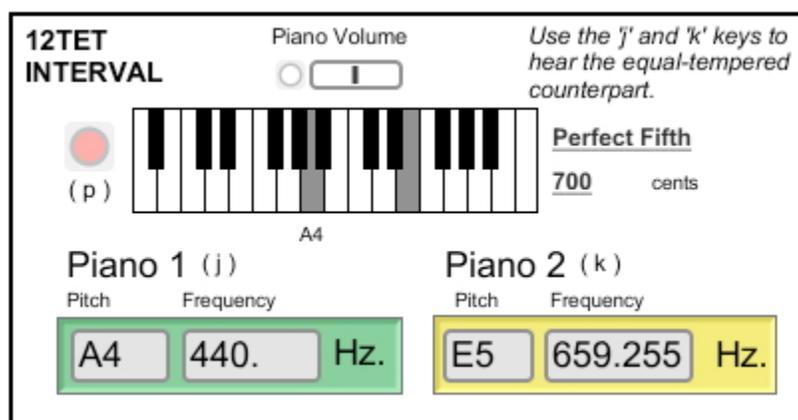
Fig. 11. The 12tet Interval Information area.

Like String 1, *Piano 1* is always set to A4, 440 Hz. Like String 2's left segment, *Piano 2* varies with the current ratio. SLAPI determines the 12tet pitch whose frequency is closest to the frequency of String 2's left segment using Max/MSP's **mtof** object.[19] The 12tet interval is graphically portrayed on a two-octave piano keyboard, and its traditional tonal interval name and size in cents are displayed to the right of the keyboard. The keyboard, which spans the range of pitches A3–C6, was created using Max/MSP's **kslider** object.[20] Please note that the location of

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[19] http://www.cycling74.com/docs/max5/refpages/max-ref/mtof.html
[20] http://www.cycling74.com/docs/max5/refpages/max-ref/kslider.html

A4 is given below the keyboard. Note too that compound intervals (other than the octave) are displayed on the keyboard as simple intervals. To play the current 12tet interval, click on the round red button to the left of the keyboard, or press the 'p' key. To play the component pitches Piano 1 and Piano 2, press the 'j' and 'k' keys, respectively.

11. Audio Control Bar

The Audio Control Bar contains two buttons:

On/Off   A toggle button which turns audio processing on and off.

Setup   Opens Max/MSP's DSP Status window to provide access to system-level audio settings such as your audio hardware and current audio driver.[21]

When you launch SLAPI, the first thing you will want to do is turn the audio on (Fig. 12).

Fig. 12. The Audio Control Bar with audio turned on.

If your audio hardware and software are correctly configured, a hybrid string-piano tone will sound to confirm that the audio is on. If you don't hear this sound, click the Setup button and refer to section 14. Troubleshooting.

12. Playback

This section provides a summary of all of SLAPI's playback features. SLAPI is primarily designed to play *ascending pitch intervals* A4–x, where x is a pitch in the A4–A5 octave and display the corresponding string length diagrams for these intervals. SLAPI can also play *descending pitch intervals* A4–y, where y is a pitch in the A4–A3 octave, however, it cannot display string length diagrams these intervals.

Playing the Intervals

The component pitches of the pitch interval *String 1–String 2* and 12tet interval *Piano 1–Piano 2* may be played simultaneously, individually, or superimposed at will using keys on the computer keyboard. String 1 and String 2 are realized using a synthetic string sample. Piano 1 and Piano 2, on the other hand, are realized using a recorded piano sample. For more information, see Technical Details (below).

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[21] http://www.cycling74.com/docs/max5/tutorials/msp-tut/mspaudioio.html

To summarize the functions of the four component pitches:

- *String 1* always sounds A4 (440 Hz.)
- *Piano 1* always sounds A4 (440 Hz.)
- *String 2*'s left segment sounds the pitch above A4, 440 Hz. that results from the current ratio.
- *Piano 2* sounds the pitch above A4, 440 Hz. that corresponds with the current ratio's 12tet counterpart.

The playback functions have been assigned to computer keys as shown in Fig. 13.

Key	Play
d	String 1
f	String 2 (left segment)
space bar	String 1 and String 2 simultaneously
j	Piano 1
k	Piano 2
p	Piano 1 and Piano 2 simultaneously

Fig. 13. Computer key assignments.

The approximate locations of these keys are given in the layout diagram shown in Fig. 14.

s	d	f	g	h	j	k	l	p

space	bar

Fig. 14. Computer keyboard location diagram (not to scale).

Other playback methods include:

In the String Length Diagrams Area (see Fig. 4)

String 1   Click on the String 1 diagram, or click the green '1' button.

String 2   Click on the String 2 diagram, or click the yellow '2' button. <u>Please note that the right segment of String 2 cannot be played</u>.

String 1 & 2   Click the round red button to play String 1 and String 2 (left segment) simultaneously. This is equivalent to pressing the space bar.

In the 12tet Area (see Fig. 11)

Piano 1 & 2   Click the round red button to play Piano 1 and Piano 2 simultaneously. This is equivalent to pressing the 'p' key.

Volume Control

      The volume of the string and piano samples may be controlled using the String Volume (Fig. 4) and Piano Volume (Fig. 11) sliders, respectively. Drag to the right to make the sound louder. Drag to the left to make the sound softer. Click the round reset buttons located to the left of the sliders to return a slider to its default value. Please note that you cannot control the volume of the component pitches independently, that is, String 1 and 2 will always be of equal volume, as will Piano 1 and Piano 2.

Technical Details

      The synthetic string sound was created with Csound's **pluck** opcode,[22] an implementation of the Karplus-Strong physical model for a plucked string.[23] The string sample is 4 seconds long with a one-second fade out added to the end of the tone. The piano sample is a modified version of a piano sample[24] available in the University of Iowa's open source Electronic Music Studio Music Instrument Sample Library.[25] The piano sample is also 4 seconds long with a one-second fade out at the end of the tone. The two samples were pitch matched using Antares' Auto-Tune Evo.[26] The string and piano samples are rendered in real-time by Max/MSP's variable-rate looping playback object **groove~**.[27] Consequently, the duration of the String 2 and Piano 2 tones will vary inversely with the current ratio. For example, if the current ratio is 2:1, the duration of String 2 tone will be 2 seconds; If the current ratio is 1:2, the duration of the String 2 tone will be 8 seconds; and so forth.

13. Messages

      SLAPI's message area confirms successful entry of the current ratio with the message:

<div align="center">Ready</div>

If the current ratio is less than 1:1 and greater than or equal to 1:2, SLAPI will issue the alert dfdf:

<div align="center">ALERT - SLAPI cannot display ratios less than 1:1.</div>

If the current ratio is greater than 2:1, SLAPI will issue the error message:

<div align="center">ERROR – Current ratio is greater than 2:1.</div>

If the current ratio is greater than 1:2, SLAPI will issue the error message:

<div align="center">ERROR – Current ratio is less than 1:2.</div>

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[22] Csound is available online at: <http://www.csounds.com>.
[23] Karplus and Strong 1983.
[24] http://theremin.music.uiowa.edu/MIS.piano.html
[25] http://theremin.music.uiowa.edu/MIS.html/
[26] http://www.antarestech.com/products/auto-tune-evo.shtml
[27] http://www.cycling74.com/docs/max5/refpages/msp-ref/groove~.html

14. Troubleshooting

If you're not hearing any sound:

- Check your computer's system sound volume level.

- Click the Setup button in the Audio Control Bar. This will open the DSP Status window. Check the settings to make sure the correct audio *driver* and *output destination* are selected.

15. Contact

Your questions, comments and suggestions are welcome. Contact the author Reginald Bain at rbain@mozart.sc.edu.

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17. Acknowledgements

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18. References

Backus, John. 1977. *The Acoustical Foundations of Music*. New York: Norton.

Bain, Reginald. 2006. *The Harmonic Series: A Path to Understanding Music Intervals, Scales, and Tuning Systems*. Available online at:
< http://www.music.sc.edu/fs/bain/atmi2002/>.

Blackwood, Easley. 1986. *The Structure of Recognizable Diatonic Tunings*. Princeton: Princeton University Press, 1986.

Cycling '74. 2010. *Max/MSP Documentation*. Available online at:
<http://www.cycling74.com/docs/max5/vignettes/intro/docintro.html>.

Doty, David B. 2002. *The Just Intonation Primer: An Introduction to the Theory and Practice of Just Intonation*, 3rd ed. San Francisco: The Just Intonation Network.

Duffin, Ross W. 2007. *How Equal Temperament Ruined Harmony (and Why You Should Care)*. New York: Norton.

Fauvel, John, Raymond Flood and Robin Wilson. 2003. *Music and Mathematics: From Pythagoras to Fractals*. New York: Oxford.

Gann, Kyle. 2006a. *Anatomy of an Octave*. Available online at:
< http://www.kylegann.com/Octave.html >.

_____. 2006b. *An Introduction to Historical Tunings*. Available online at:
< http://www.kylegann.com/histune.html >.

_____. 2006c. *Just Intonation Explained*. Available online at:
< http://www.kylegann.com/tuning.html >.

_____. 2006d. *My Idiosyncratic Reasons for Using Just Intonation*. Available online at:
< http://www.kylegann.com/JIreasons.html>.

Campbell, Murray and Clive Greated. 1994. *The Musician's Guide to Acoustics*. New York: Oxford University Press.

Karplus, Kevin, and Alex Strong, 1983. "Digital Synthesis of Plucked-String and Drum Timbres" in *Computer Music Journal*, Vol. 7, No. 2, Summer 1983. Reprinted in the *Music Machine* (MIT Press).

Loy, Gareth. 2006. "Musical Scales, Tuning, and Intonation." In *Musimathics: The Mathematical Foundations of Music*, 39-96. Cambridge, MA: The MIT Press.

Partch, Harry. *Genesis of a Music*, 2nd ed. New York: Da Capo, 1974.

Roederer, Juan G. 2008. *The Physics and Psychophysics of Music: An Introduction*, 4th ed. New York: Springer.

Weisstein, Eric W. 2010. *MathWorld--A Wolfram Web Resource*. Available online at:
< http://mathworld.wolfram.com>

19. Appendix 1

The Enter Ratio menu contains the following 60 preprogrammed pitch intervals.

1. Unison 1:1	31. Small septimal tritone 7:5
2. Syntonic comma 81:80	32. Large just tritone 45:32
3. Diminished second 128:125	33. Small just tritone 64:45
4. ET quarter-tone approx. 35:34	34. Large septimal tritone 10:7
5. Small just minor second 25:24	35. Diminished fifth 36:25
6. Pythagorean half-step 256:243	36. Wolf fifth 40:27
7. ET minor second approx. 18:17	37. Just perfect fifth 3:2
8. Overtone half-step 17:16	38. Diminished sixth 192:125
9. Large just minor second 16:15	39. Septimal minor sixth 14:9
10. Cowell just half-step 15:14	40. Augmented fifth 25:16
11. Undecimal half-step 12:11	41. Undecimal minor sixth 11:7
12. The small just major second 10:9	42. Pythagorean minor sixth 128:81
13. The large just major second 9:8	43. Just minor sixth 8:5
14. Septimal major second 8:7	44. Overtone sixth 13:8
15. Diminished third 144:125	45. Undecimal sixth 18:11
16. Septimal minor third 7:6	46. Just major sixth 5:3
17. Augmented second 75:64	47. Pythagorean major sixth 27:16
18. Pythagorean minor third 32:27	48. Diminished seventh 128:75
19. Overtone minor third 19:16	49. Septimal major sixth 12:7
20. Just minor third 6:5	50. Augmented sixth 125:72
21. Undecimal minor third 11:9	51. Septimal minor seventh 7:4
22. Just major third 5:4	52. Small just minor seventh 16:9
23. Pythagorean major third 81:64	53. Large just minor seventh 9:5
24. Diminished fourth 32:25	54. Undecimal seventh 11:6
25. Septimal major third 9:7	55. Small just major seventh 15:8
26. Augmented third 125:96	56. Pythagorean major seventh 243:128
27. Septimal fourth 21:16	57. Large just major seventh 48:25
28. Just perfect fourth 4:3	58. Septimal major seventh 27:14
29. Undecimal tritone 11:8	59. Augmented seventh 125:64
30. Augmented fourth 25:18	60. Octave 2:1

Click on the Enter Ratio menu and select one of the ratios to make it the current ratio. The interval names were derived from Partch 1971 and Gann 2006a.