

## CHAPTER 3

### *The Frequency Domain*

“French scientist and mathematician Jean Baptiste Fourier (1768-1830) proved...that any periodic waveform can be expressed as the sum of an infinite set of sine waves. The frequencies of these sine waves must be integer multiples of some fundamental frequency.”

– Burk et al., *Music and Computers*

### Terms & Concepts

<p><b>3.1 Frequency Domain</b></p> <p>Domains</p> <ul style="list-style-type: none"> <li>- Time (<i>a</i> vs. <i>t</i>)</li> <li>- Frequency (<i>a</i> vs. <i>f</i>)</li> <li>- Amplitude</li> </ul> <p>Envelopes</p> <ul style="list-style-type: none"> <li>- Transients <ul style="list-style-type: none"> <li>+ Attack stage</li> <li>+ Decay stage</li> <li>+ Release stage</li> </ul> </li> <li>- Steady state stage</li> <li>- Average signal envelope</li> <li>- Peak signal envelope</li> </ul> <p>Root-mean-squared (RMS) amplitude</p> <p>Running window technique</p> <p>Sonogram (<i>f</i> vs. <i>t</i>)</p> <p>Melograph (<i>pitch</i> vs. <i>t</i>)</p> <p>Phonophotography</p> <p><b>3.2 Phasors</b></p> <p>Phasor representation of a sine wave</p> <p>Sound analysis</p> <p>Digital manipulation of sound</p> <p>Sine wave model</p> <p>Phasor function</p> <p>Trigonometric functions</p> <p>Degrees</p> <p>Radians</p> <p>Angular velocity of the phasor</p> <p>Law of superposition (adding phasors)</p> <p>Fourier’s theorem</p> <p>Periodic function</p> <p>Fundamental frequency</p> <p>Overtones, partials and harmonics</p>	<p>Odd-partial symmetry</p> <p>Vectors</p> <ul style="list-style-type: none"> <li>- Magnitude</li> <li>- Direction</li> </ul> <p>Vector addition</p> <p><b>Sampling and Fourier Expansion</b></p> <p>Fourier expansion</p> <p>Fourier coefficients</p> <p>Bins</p> <p>Waterfall 3D plot</p> <p><b>3.3 Fourier and the Sum of Sines</b></p> <p>Jean Baptiste Fourier (1768-1830)</p> <p>Basic waveshapes</p> <ul style="list-style-type: none"> <li>- Sine</li> <li>- Sawtooth</li> <li>- Square <ul style="list-style-type: none"> <li>+ Pulse (Duty cycle)</li> </ul> </li> <li>- Triangle</li> </ul> <p>Complex waveform</p> <p>Spectrum</p> <p>Infinite series</p> <p>Fourier series</p> <p>Fourier analysis, synthesis and transform</p> <p>Fourier coefficients</p> <ul style="list-style-type: none"> <li>- Low order</li> <li>- High order</li> </ul> <p>DC term</p> <p>Filters</p> <ul style="list-style-type: none"> <li>- Low pass</li> <li>- High pass</li> <li>- Band pass</li> </ul> <p>Hydrophone</p>	<p>Fourier analysis</p> <p><b>3.4 The DFT, FFT, and IFFT</b></p> <p>Discrete Fourier Transform (DFT)</p> <p>Fast Fourier transform (FFT)</p> <p>Inverse Fast Fourier transform (IFFT)</p> <p>Sample rate</p> <p>Frame size (as a power of 2)</p> <p>Number of bins</p> <p>Bin width</p> <p>Windowing</p> <p>Histogram of frequencies</p> <p style="text-align: center;"><i>number of bins = Frame size / 2</i></p> <p style="text-align: center;"><i>bin width = f range / # of bins</i></p> <p><b>3.5 Problems with the FFT/IFFT</b></p> <p>Time/frequency resolution trade-off</p> <p>Lobes</p> <p>Time smearing</p> <p><b>3.6 Some Alternatives to the FFT</b></p> <p>Wavelet analysis</p> <p>McAulay-Quatieri (MQ) Analysis</p> <p><b>Software</b></p> <p>Faber Acoustic’s <i>SignalScope</i></p> <p>Apple’s Grapher</p> <p>Wolfram’s Mathematica</p>
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**Reference**

Burk, Phil, Larry Polansky, Douglas Repetto, Mary Roberts and Dan Rockmore. 2011. *Music and Computers: A Theoretical and Historical Approach*, Archival Version. Available online at: <http://music.columbia.edu/cmcc/MusicAndComputers/>.